DETERMINE THE OPTIMAL NUMBER OF SPARE PARTS USING FUZZY QUEUING MODEL WITH LIMITED SOURCE

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Abstract

Queuing models with limited source have many applications in practical situations and seen frequently in operation and maintenance of vehicles. Due to uncontrollable factors, parameters of queuing issues with limited resource may be inconsistent. So the fuzzy queuing model with finite resource can be closer to reality. In this article in order to design the system and determine the optimal number of spare parts, we seek to minimize the total cost function while the repair rate and the input rate are fuzzy numbers; also the cost function includes the cost of maintenance spare pieces and the cost of system deactivation. After calculating the value of cost function for different number of spare parts, we compare the value of the obtained function by using Liou -Wang and Nakamura methods and concluded that the keeping of how many spare parts are economical.

Key Words: Queuing Model, Fuzzy Theory, Limited Input Source, Fuzzy Cost Function

1. Introduction

Basic queuing models based on the assumption that the input sources are infinite and inputs are controlled by a Poisson process. Unlike these models, the queuing model with limited source $M/M/c/k$, the inputs depends on the within units of it. Very important application of

these models is in the maintenance of vehicles.

Maintenance responsibility of each group of machines is allocated to one or more repairmen. Each machine is considered as part of the queuing system which is being repaired or waiting for repair, while outside of system is active and is working. This model has many applications in production system, industrial and maintenance operation systems. In former queuing theory, the time between the inputs and the length of repair were determined by the probability distribution, But in many situations Statistics are obtained subjectively. So the pattern of input and service are described by linguistic terms such as fast, slow. For this reason fuzzy models are used more than classical models.

Although fuzzy models are more functional but relatively fewer studies have been done on them. For example, Buckley et al. [1] consider multi-server queuing system with finite or infinite capacity which inputs follow a feasible pattern. Buckley et al. [2] expand the results of this model for fuzzy models. Chen [3] offers parametric programming for making the membership function of queuing decision problems. Chollette et al [4] provide an algorithm based on fuzzy logic for scheduling queues. Kumar [5] uses a nonlinear programming approach to establish membership functions in Erlang model in which the input and service

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rates are fuzzy. Zafari [6] applies fuzzy control approach to reduce the cost of queuing system with fuzzy service rate. Fu [7] offers a method for estimating waiting vehicles in the queue based on fuzzy logic in city junctions.
This article analyses the queuing model with finite source based on the classic models which is developed by Kleinrock [8], Hillier and Lieberman [9], Gross and Harris [10] and Taha [11]. Pardo and Fuente [12] examine fuzzy queuing model with limited resource to optimize the number of servers.
This article includes the following sections: In Section 2 the classical queue model will be described with limited resource and the expected total cost function will be calculated for the y spare parts. Then, using the extension principle of Zadeh [13] fuzzy queuing model will be developed when the parameters λ and μ are fuzzy and the cost function will be written. In the following two methods of Liou and Wang [14] and Nakamura [15] will be described to compare maintenance cost of different spare parts. In section 3, a case study will describe in the plastic injection industry and related calculations will be performed.

2. Fuzzy Queuing Model with Limited Source $M/M/c/m$

In the basic queuing models it is assumed that the inputs come from the population (the demander's population) are unbounded because the number of entries in each input is a Poisson random variable which have the unlimited size of sample space. In the real world usually it is not like this.
For instance, in maintenance systems at industrial centers, Inputs are the cars that are likely to deteriorate and limited in numbers. Now we consider a problem with limited population of demanders for example, the population of applicants is equal to *m* and probabilities of events will be the function of past behavior. A typical application for this model is car servicing where the demanders population are the cars or parts of cars that have become damaged and require repair. Repairmen are the service providers.
We assume that c servers (repairmen) are available and Service times are identical random exponential variable with the mean of 1/μ (probability of selecting servers are same). Suppose we have n spare units, So that when a machine will fail, it is immediately replaced by a spare piece. If in the event of mechanical failure, all spare parts are in use, the system is disconnected.
Also when the car will be repaired will turn to spare car unless the system is disconnected in which case the repaired car will be immediately entered to service, and will be used. Now we want to achieve the optimal number of spare machines so that the costs will be minimized. For this model λ_n (the rate of machine failure) is as following [10]:

$$\lambda_n = \begin{cases} m\lambda & 0 \le n < y \\ (m - n - y) & y \le n < y + m \\ 0 & n \ge y + m \end{cases} \tag{1}$$

Consider C repairmen. μ_n (service rate) is equal:

$$\mu_n = \begin{cases} n\mu & if\ n = 0,1,\ldots,s \\ c\mu & if\ n = c, c+1\ldots \end{cases} \tag{2}$$

The probability of n units in the system will be obtained of the following relationships:
First of all assume $c \le y$:

$$p_n = \begin{cases} \frac{m^n}{n!}\left(\frac{\lambda}{\mu}\right)^n p_0 & 0 \le n < c \\ \frac{m^n}{c^{n-c}c!}\left(\frac{\lambda}{\mu}\right)^n p_0 & c \le n < y \\ \frac{m^y m!}{(m-n+y)!\, c^{n-c}c!}\left(\frac{\lambda}{\mu}\right)^n p_0 & y \le n < y+m \end{cases} \quad (3)$$

If y is very large, essentially we will have the unlimited number of input demanders with average rate of $m\lambda$ and the results of the *m/m/c/∞* model is obtained which mλ is replaced as λ

[10].

When *c>y* the p_n (probability of n people in the system) can be calculated from the following

equation:

$$p_n = \begin{cases} \frac{m^n}{n!}\left(\frac{\lambda}{\mu}\right)^n p_0 & 0 \le n < y \\ \frac{m^\lambda m!}{(m-n+y)!\, n!}\left(\frac{\lambda}{\mu}\right)^n p_0 & y+1 \le n < c \\ \frac{m^y m!}{(m-n+y)!\, c^{n-c}c!}\left(\frac{\lambda}{\mu}\right)^n p_0 & c \le n < y+m \end{cases} \quad (4)$$

With constraint:

$$\sum_{n=0}^{y+m} p_i = 1 \quad (5)$$

We want to obtain the optimal number of spare parts which the cost of expected maintenance of spare parts and system disability will be minimized. For this purpose we introduce the following costs:

s_1*: Maintenance cost of each spare part per unit of time*

s_2*: System disability cost per unit of time*

E_{y1}*: The function of expected maintenance cost for y spare parts per each unit of time*

E_{y2}*: The function of expected system disability cost for the y spare parts per each unit of time*

$E(TC)_y$*: The function of expected total cost of system for y Spare Parts per each unit of time*

$$E_{y1} = \begin{cases} s_1 V p_0 & n = 0 \\ \sum_{n=1}^{y} s_1 (y-n) p_n & 1 \le n \le y \end{cases} \quad (7)$$

$$E_{y2} = s_2 \sum_{n=y+1}^{m+y} p_n \qquad y+1 \le n \le m+y \quad (8)$$

$$E(TC)_y = E_{y1} + E_{y2} \quad (9)$$

$$E(TC)_y = s_1 Y p_0 + \sum_{n=1}^{y} s_1(y-n)p_n + s_2 \sum_{n=y+1}^{m+y} p_n \tag{10}$$

In this model we are seeking to optimization solution of this problem

$$Min\, E(TC)_y = E_{y1} + E_{y2} \tag{11}$$

Now we consider the case that the repair rate λ and service rate μ is not conclusive and we are dealing with the fuzzy numbers $\tilde{\lambda}$ *and* $\tilde{\mu}$. Membership functions of $\tilde{\lambda}$ *and* $\tilde{\mu}$ is defined for us.

By applying the development principle of Zadeh [13], we can get the fuzzy possibilities membership function. s_1 *and* s_2 can be deterministic or fuzzy. In this model they were considered deterministic. Generally we have:

$$\tilde{E}(TC)_y = s_1 y \tilde{P}_0 + \sum_{n=1}^{y} s_1(y-n)\tilde{p}_n + s_2 \sum_{n=y+1}^{m+y} \tilde{p}_n \tag{11}$$

Optimal amount of fuzzy queuing model with finite source and c server is the number of spare parts which minimized fuzzy expected total cost per unit of time.

$$Min \tilde{E}(TC)_y = s_1 y \tilde{P}_0 + \sum_{n=1}^{y} s_1(y-n)\tilde{p}_n + s_2 \sum_{n=y+1}^{m+y} \tilde{p}_n \tag{12}$$

In general, all functions for $\tilde{\lambda}$ and $\tilde{\mu}$ parameters based on the principle of Zadeh [10] (λ input rate to support of $\tilde{\lambda}$ and μ support service rate belongs to the support of $\tilde{\mu}$) are defined as follows:

$$\mu_{f(\tilde{\lambda},\tilde{\mu})} = sup\left\{\mu_{\tilde{\lambda}}(\lambda) \wedge \mu_{\tilde{\mu}}(\mu) / z = f(\lambda, \mu)\right\} \tag{13}$$

To solve this problem, we present two techniques for decision making under uncertainty.

1. Liou and Wang method [14]: they introduced a method for selecting the alternatives by using the "integral amount". This method can simultaneously compare more than two alternatives. $A_\alpha = \lfloor \underline{A}_\alpha, \bar{A}_\alpha \rfloor$ is cut of α from $\pi_A$and $B_\alpha = \lfloor \underline{B}_\alpha, \bar{B}_\alpha \rfloor$ is from π_B. The left integral value of A is defined as follows:

$$I_L(A) - \int_0^1 \underline{A}_\alpha \, d\alpha \tag{14}$$

And the right integral value:

$$I_R(A) = \int_0^1 \overline{A}_\alpha \, d\alpha \tag{15}$$

The total integral value with optimal index of α is defined as follows:

$$I_T^\alpha(A) - \alpha I_R(A) + (1-\alpha) I_L(A) \qquad \alpha \in [0,1] \tag{16}$$

Total integral value of A is used as a ranking function. So for each alternatives of A and B if $I_T^\alpha(A) < I_T^\alpha(B)$ *then* $A < B$*. If* $I_T^\alpha(A) = I_T^\alpha(B)$ *then* $A = B$*, if* $I_T^\alpha(A) > I_T^\alpha(B)$*then* $A > B$*. The value of* α *is determined based on the optimization degree of decision maker.*

2. Nakamura *method [15]: Nakamura defined* Fuzzy preference relation $\mu_N(\tilde{A},\tilde{B})$ for alternatives of A and B with below membership function.

$$\mu_N(\tilde{A},\tilde{B}) = \begin{cases} \dfrac{(1-\beta)T_1 + \beta T_3}{T_\beta} & if\ T_\beta \neq 0, \\ 1/2 & if\ T_\beta = 0, \end{cases} \quad \beta \in [0,1] \tag{17}$$

In which

$$T_\beta = (1-\beta)(T_1 + T_2) + \beta(T_3 + T_4)$$

$$T_1 = \int_{\{\alpha/\underline{A}_\alpha > \underline{B}_\alpha\}} [\underline{A}_\alpha - \underline{B}_\alpha] d\alpha$$

$$T_2 = \int_{\{\alpha/\underline{A}_\alpha < \underline{B}_\alpha\}} [\underline{B}_\alpha - \underline{A}_\alpha] d\alpha$$

$$T_3 = \int_{\{\alpha/\bar{A}_\alpha > \bar{B}_\alpha\}} [\bar{A}_\alpha - \bar{B}_\alpha]\, d\alpha$$

$$T_4 = \int_{\{\alpha/\bar{A}_\alpha < \bar{B}_\alpha\}} [\bar{B}_\alpha - \bar{A}_\alpha] d\alpha \tag{18}$$

The parameter β indicates the attitude of the decision maker at risk. If you feel risk then β is larger than 0.5. In the neutral situation β = 0.5.

If $\mu_N = 1/2$, then A and B do not matter. If $\mu_N > 1/2$, then A is preferred to B. If $\mu_N < 1/2$, then B is preferred to A.

3. Case Study

This technique was used for a plastic injection machine in a factory. In this factory each plastic injection machine is used to produce several different types of plastic parts. For the production of each piece it is necessary to install the molds of that piece.

If the mold is broken, the production line will be stopped. One of the pieces produced by this machine and non-production or delay in production of it caused a lot of damages to the plant, is bowl of motorcycle light.

For produce of this piece, there are four same pieces in case of deterioration of one of them; the other one is used to replace existing part. The failure rate is constant and equal for all units (including molds are quite similar).

Tow repairmen were also considered. We consider failure rates and repair rates as triangular fuzzy number, $\tilde{\lambda} = [0.25, 0.33, 0.4]$*and* $\tilde{\mu} = [1.5, 2, 3]$. *The maintenance cost of each*

spare unit (mold) for each definite day is 2200 currency and the failure cost for a definite date is equal to 19,000 currency (For simplicity, we ignore the three zero).

The purpose is determining the number of components (molds) required to optimize the fuzzy queue model. In fact, we consider that keeping which number of spare pieces (mold) is more economical. Considering that $\alpha = 0.5$ and $\beta = 0.5$, we solve the problem. We consider fuzzy

cost function that must be minimized (membership functions are obtained by using Zadeh[13] extension principle)

$$\tilde{E}(TC)_y = s_1 y \tilde{P}_0 + \sum_{n=1}^{y} s_1 (y-n) \tilde{p}_n + s_2 \sum_{n=y+1}^{m+y} \tilde{p}_n$$

To check the two conditions we consider y = 1 and 2 because when y is greater than 2, the expected total cost is more than these two states. Membership functions are drawn in Figure 1. (Membership functions of these two states overlap, and in other cases it is more expensive and does not need to check.)

- ***The model with one spare unite,*** $c > y, y = 1$*:*

$$\tilde{E}(TC)_1 = 2.2\tilde{p}_0 + 19\,\tilde{p}(n \geq 2) \qquad (19)$$

$$\tilde{p}_n = \begin{cases} \dfrac{4^n}{n!}\left(\dfrac{\tilde{\lambda}}{\tilde{\mu}}\right)^n \tilde{p}_0 & 0 \leq n \leq 1 \\ \dfrac{4 * 4!}{(5-n)!\,2^{n-2}\,2!}\left(\dfrac{\tilde{\lambda}}{\tilde{\mu}}\right)^n \tilde{p}_0 & 0 \leq n \leq 5 \end{cases}$$

We use the above formula to calculate the value of $\tilde{p}_i$ *then we obtain the cost function.*

- *The model with tow spare unites* $c \leq y, y = 2$ *:*

$$\tilde{E}(TC)_2 = 2.2 * 2 * \tilde{p}_0 + 2.2\sum_{n=1}^{2}(2-n)\tilde{p}_n + 19\,\tilde{p}\,(n \geq 3) \qquad (20)$$

$$\tilde{p}_n = \begin{cases} \dfrac{4^n}{n!}\left(\dfrac{\tilde{\lambda}}{\tilde{\mu}}\right)^n \tilde{p}_0 & 0 \le n < 2 \\ \dfrac{4 * 4!}{(6-n)!\,2^{n-2}\,2!}\left(\dfrac{\tilde{\lambda}}{\tilde{\mu}}\right)^n \tilde{p}_0 & 2 \le n \le 6 \end{cases}$$

We use the above formula to calculate the value of $\tilde{p}_i$ *then we obtain the cost function. The expected total cost of fuzzy membership functions are drawn in Figure 1. To determine the number of spare parts to minimize the total cost function, we compared the cost per each day.*

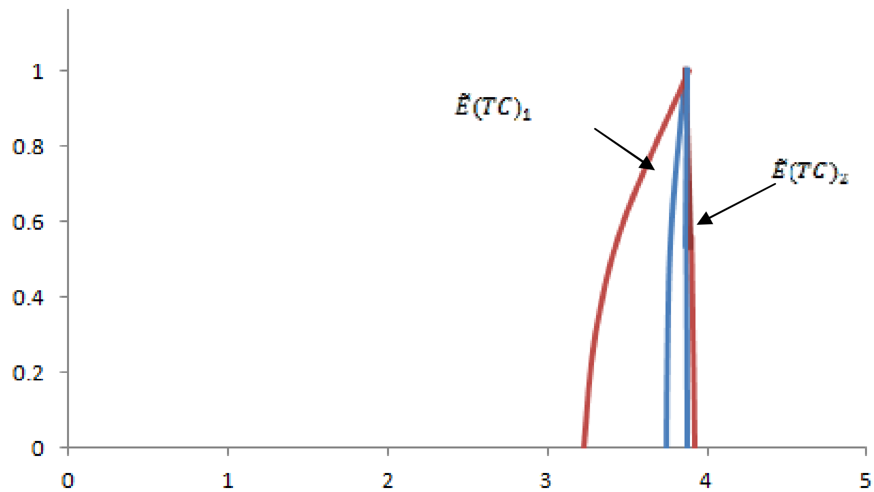

Figure 1. fuzzy expected total cost function

1. Liou and Wang method [14]: the total integral value with optimized parameters α = 0.5 for each function of the fuzzy expected cost per a day is as follows:

$$I_T^{0.5}\tilde{E}(TC)_2 = 0.5 I_R\tilde{E}(TC)_y + (1-0.5) I_L\tilde{E}(TC)_y \tag{21}$$

$$I_R\tilde{E}(TC)_y = \int_0^1 \overline{E}_\alpha(TC)_y\, d\alpha\,,\quad I_L\tilde{E}\tilde{E}(TC)_y = \int_0^1 \underline{E}_\alpha(TC)_y\, d\alpha \tag{22}$$

We have obtained the following results:

$I_R\tilde{E}(TC)_1 = 3.905085$

$I_L\tilde{E}(TC)_1 = 3.559084$

$$I_T^{0.5}\tilde{E}(TC)_1 = 0.5 \times (3.905085 + 3.559084) = 3.7320845 \tag{23}$$

$I_R\tilde{E}(TC)_1 = 3.8838458$

$I_L\tilde{E}(TC)_1 - 3.8074975$

$$I_T^{0.5}\tilde{E}(TC)_2 = 3.838458 \quad (24)$$

With this method $\tilde{E}(TC)_1 < \tilde{E}(TC)_2$. So when we have one spare piece the system is efficient.

2. Nakamura method [15]: Fuzzy preference relations $\mu_N(\tilde{E}(TC)_1, \tilde{E}(TC)_2)$ and $\mu_N(\tilde{E}(TC)_2, \tilde{E}(TC)_1)$ were calculated and the results are shown as below.

$$\mu_N(\tilde{E}(TC)_2, \tilde{E}(TC)_1) = 0.873$$

$$\mu_N(\tilde{E}(TC)_1, \tilde{E}(TC)_2) = 0.093$$

Table 1. Fuzzy relation matrix

μ_N	$\tilde{E}(CT)_1$	$\tilde{E}(CT)_2$
$\tilde{E}(TC)_1$	**0.5**	**0.093**
$\tilde{E}(TC)_2$	**0.873**	0.5

Fuzzy relation matrix is created in the table above. Fuzzy relations matrix between the different values of the fuzzy expected total cost allows clustering between them: the row that all of its elements are equal or smaller than 0.5, is the best choice. Then the row and column indicate the best choice has been removed and the same reasoning applies to the remaining alternatives.

Therefore with Nakamura method the order of precedence is as follows:

$$\tilde{E}(CT)_1 < \tilde{E}(CT)_2 \quad (25)$$

By selecting the optimal service level, other measures of system performance can be achieved.

Fuzzy comparison of both methods shows having one spare piece is more affordable. Therefore it seems that a spare part can reduce the cost of overall system.

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