



Research Paper

# Validity of NPV rule and IRR criterion for Capital Budgeting and Cost Benefit Analyses

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## ABSTRACT

The validity of the NPV and IRR rules is evaluated and presented. With normal net cash flow:

a. The NPV and IRR estimates lead to the same decision, b. Neither the NPV nor the IRR always involves reinvestment; c. reinvestment might make the NPV spurious with low discount rates, but the IRR remains unique. With nonnormal net cash flow: a. spurious NPVs and multiple IRRs coexist; b. A capital amortisation method eliminates reinvestment and resolves the problems of multiple IRR and spurious NPV and enables estimation of IRR on total capital and or balance of capital. c. IRR ranks consistently more than NPV.

## 1 | INTRODUCTION

The 'NPV rule' dominates in CBA, capital budgeting and investment analysis. The problem of multiple or negative or no IRR is universally regarded as a fatal flaw for the IRR estimate (see Hazen, 2003; Kierulff 2011; Osborne 2010; Brigham et al. 2012). According to Merlo (2012), with non-monotonic<sup>2</sup> NPV function originating from non-normal NCF (NNCF)<sup>3</sup> Using the NPV rule as if NPV is without errors and abandoning the IRR is incorrect. Merlo also concluded that 'treating disease symptoms when necessary to diagnose its causes' is wrong. Illes (2020) studied the loss-making investments with positive NPV and multiple IRR and concluded that the economic content of a positive net present value is false when the investments lead to net loss. These conceptual problems remain unresolved despite several attempts by many authors. These unresolved issues are carefully evaluated and addressed. With a normal NCF, the IRR criterion will make the same NPV decision. Theoretically, both NPV and IRR are estimated by solving the same NPV function for a desired hurdle rate (cost of capital (CoC) or weighted average cost of capital (WACC)) and for an unknown rate, respectively. Any problem with NNCF must equally adversely affect both IRR and NPV. Applying the NPV rule without further investigating the real problem is a bounded rationality<sup>4</sup> approach that might lead to suboptimal decision-making. Such ongoing serious criticisms would continue to undermine the value of CBA and its application in capital budgeting, valuation and regulatory analysis. Under that context, the present analysis is important to evaluate the robustness of the NPV rule and the validity of controversies surrounding the CBA criteria (IRR or NPV) and to inform the analysts and decision makers appropriately. Other commonly prevailing arguments to justify the NPV rule are that the IRR is not suitable for ranking and selecting mutually exclusive projects or projects with multiple IRR (e.g., Osborne 2010, Kierulff 2012, World Bank 2013, Asian Development Bank (ADB) 2017, Juhász 2011). Many texts also discuss that IRR involves reinvestment at a rate equivalent to IRR, whereas NPV only reinvests at the cost of capital.

The findings presented here make six important contributions: i. identify and quantify the reinvestment, if any, that causes the coexistence of spurious NPV and multiple IRR; ii. the notion of reinvestment at cost of capital or at IRR is not always true; iii. Elimination of reinvestment mostly associated with NNCF, if any, using a Capital Amortization Schedule (CAS) approach and resolving the problem of spurious NPV and multiple IRR; and iv. demonstrate that the NPV rule is not always valid and the criticism against IRR is also not valid; and v. IRR ranks mutually exclusive investments consistently better than the NPV; and vi The DCF method estimates the IRR on the balance of capital whereas the Capital Amortization Schedule (CAS)

method enables estimation of IRR on total capital investment or on balance of capital.

## 2 | LITERATURE REVIEW

There are three criticisms against the CBA method: a. the validity of the DCF method; b. criticism of the IRR criterion and preference of NPV rule; and c. criticism that IRR is not the return on the total capital but on the remaining capital. Seven decades ago, Lorie and Savage (1956) considered the DCF method for CBA as "ambiguous or anomalous and concluded that with such an ambiguity "the rate-of-return solution does not result in maximising the present worth of the firm's net worth." In litigations involving CBA in the USA and Australia, some judgements question the validity of the analysis (for example, Conn, 2013; Albany & Ors v The Commonwealth of Australia 1976). Merlo (2016) discussed that the popular DCF method is not appropriate for estimating NNCF, and the estimated IRR and NPV are incorrect. There are criticisms that IRR is not the return on the total capital but on the remaining capital or the declining balance of the investment (Kelleher and MacCormack 2004, Rich and Rose 2015, Dudley 1972). IRR and NPV are currently being estimated using the well-known discounted cash flow (DCF) method. A CAS approach to CBA, discussed by Arjunan (2019), is used along with the DCF method to study the validity of these problems and to resolve them.

The World Bank (2013) and the Asian Development Bank (ADB, 2017), in their Guidelines on Investment Project Analysis, listed the problems of ranking mutually exclusive investments and multiple IRR and preferred using NPV in those cases. Weber (2014) reported that the IRR is generally considered inferior to the net present value (NPV) as a tool for evaluating and ranking projects. Cuthbert and Cuthbert (2012) argued that IRR is potentially misleading. Some others concluded that IRR is a limited decision tool and advised the financial analysts to use it cautiously (see also Kierulff 2012 and Kelleher 2004). Phalippou (2009) stated that IRR has its own shortcomings and biases and creates room for managers to manipulate the performance reporting. Park (2005) recommended abandoning the IRR analysis and using the NPV criterion whenever analysts encounter multiple rates of return. With NNCF investments, the estimated IRR may be zero, negative or multiple (see also Ben-Horin and Kroll 2012; Arjunan 2019). Ross (1995) did a detailed analysis and concluded that 'for most investments, the usefulness of NPV rule is severely limited' where an investment does not preclude alternative investment options.

Weber (2017) introduced a selective IRR (SIRR) criterion that he claimed to be equivalent to the NPV rule. He argued that "an investor with a cost of capital of  $r = 10\%$  would report the return of a project with the cash-

<sup>1</sup> The NPV rule is to invest in projects that have a highest positive net present value (NPV).

<sup>2</sup> A NPV function is non-monotonic when it increases as well as decreases at intervals with increasing discount rates.

<sup>3</sup> Normal Net cash flow (NCF) change signs only once. It starts with negative cash flows during the first year or first few years (investment years) followed by only positive NCFs for all the subsequent years. Non-normal NCF changes signs more than once with negative intermediate cashflows.

<sup>4</sup> For definition see Simon 1982.

flow stream (-5, 16, -12) as minus infinity (and therefore completely unacceptable), whereas an investor with a cost of capital of  $r = 25\%$  would report the return of the same cash-flow stream as 100% which is very attractive indeed.” As could be seen, when the cumulative undiscounted sum of the above NCF is -1 (cumulative loss), the IRRs and NPVs are spurious. That being the case, a negative or zero IRR is possible. A 100% return is highly unrealistic and purely a symptom associated with NDCF that is not supported by the project benefit stream (sum of NCF = -1, i.e., net loss). Some authors reported that IRR and NPV guarantee making relevant decisions (for example, Juhász, 2011, Hall and Millard, 2010). Jacobs (2007) argued that ‘the conflict between NPV and IRR arises because of misinterpretations that have been made. The NPV-method and IRR methods are not two measures of investment worth - as reported in many textbooks - but just one method. His argument is correct because the same NPV function is used to estimate NPV and IRR by the DCF method. NPV is estimated by solving the function for a known discount rate, whereas IRR for an unknown discount rate that makes the NPV = 0. NPV and IRR are mathematically interrelated (IRR is the discount rate that makes NPV Zero).

Berkovitch and Israel (2004) showed that using NPV as an investment criterion leads to inefficient capital budgeting outcomes; therefore, other capital budgeting criteria, like the IRR and the profitability index (PI), will continue to dominate. Hazen (2003) reported that the problem of multiple or non-existent IRR is not a flaw and can be easily dealt with conceptually and procedurally. Several authors believe that multiple internal rates constitute a severe drawback and not helpful (Brealey and Myers 1996, Sullivan et al. 2000, and Eschenbach 1995). Arjunan (2019) studied the properties of non-monotonic NPV function associated with NDCF investments and concluded that such NPV function leads to spurious NPV and multiple IRR. Arjunan (ibid) also presented a CAS approach to resolve these problems.

Some authors argue that these conflicts are caused by the problems of scale of investment, timing of cash flow, different maturity or life of investment (for example Barney and Danielson 2004). These are not problems but are associated with the nature of the investment and cash flow. Given the timing of NCF, the most profitable investment must be identified. Barney and Danielson (2004) compared mutually exclusive projects and concluded that differences in return duration cause ranking conflicts between NPV and IRR. They proposed considering duration or generalised NPV before making investment decisions when facing ranking conflicts.

In summary, IRR and NPV are criteria estimated by the same DCF method with the time value of money concept, using the same NCF data, and both estimates are mathematically interrelated. From an analytical or mathematical perspective, NPV is a point estimate (at hurdle rate). In contrast, the IRR estimate involves evaluating a range or profile of NPVs at various discount rates to locate the discount rate that makes the NPV = 0. Instead of a point estimate (NPV), if an NPV profile review is undertaken, the adequacy or inadequacy of NPV could be better explained and appreciated. This paper investigates and presents numerical evidence to better appreciate the capital investment criteria, NPV and IRR.

**3 | METHODOLOGY**

The CAS (commonly used by Banks and financial institutions) method used in this analysis is briefly discussed. CAS is a table or chart showing how much of each periodic future income or return from an investment is going towards interest payments or return (ROIC) and capital (principal) recovery (ROC). CAS analysis will indicate when the net cash inflow (NCF) is fully utilised (closing balance zero) to recover the ROC and a ROIC (IRR). The PV of the final closing balance is the NPV (see Arjunan 2019) that will be 0 when the ROIC is equal to IRR or >0 when the ROIC < IRR (underestimate) or <0 when the required ROIC > IRR (unachievable). CAS estimation follows equation 1.

$$\sum_{t=1}^n (OB_t * (1 + R) - NCF_t) = CB_t \quad \text{Equation 1}$$

Where OB is the opening balance of capital each year, ‘r’ is the interest rate (or return r), NCF is the net cash inflow or the intermediate net income per year, CB is the closing balance each year and ‘t’ is year 1 to n (n is terminal or final year of project life or maturity of investment). The CB of a year is the OB of the next year. The final year CB<sub>t</sub> is the balance of the CF<sub>0</sub>. The interest rate that makes the CB<sub>t</sub> = 0, is the IRR. When the cost of capital or WACC is used as interest, the CB<sub>t</sub> reveals the remaining balance of CF<sub>0</sub> (0 at IRR or >0 when the interest rate is below IRR or <0 if the interest rate is higher than IRR). The PV of CB<sub>t</sub> will be ‘0’ at IRR, >0 if the interest rate is below IRR and

< 0 when the interest is above IRR. Accordingly. The PV of the CB<sub>t</sub> is the NPV as below:

$$PVCB = \frac{CF_1}{(1+r)^1} - CF_0 = NPV \quad \text{Equation 2}$$

The IRR can be estimated by using the “Goal Seek” command in Excel (click data, then click What-if Analysis) with “Set cell” = final CB cell; “To value” = ‘0’; and “By changing cell” = Cell containing interest rate. This command will identify the rate that makes the final CB zero (that is IRR). The advantage of the CAS method is that it transparently reveals that there is reinvestment with NDCF. If there is reinvestment, that must be eliminated. A modified CAS method (MCAS) is designed to eliminate reinvestment and estimate the IRR and NPV for NDCF investments. The simple modification in the CAS is through ‘r’. The ‘if function’ in the excel is used to apply interest or discount at ‘r’ (r\*1) if the OB is negative and if the OB is positive then the ‘r’ is zero (r\*0). Eliminating reinvestment, which is not investment-generated income, will perfect the estimates. In the next section, the Modified CAS (MCAS<sup>5</sup>) method is explained with some worked-out examples so that analysts or economists can easily relate it to the real-life situation. CAS method is like the popular loan amortization schedule.

**4 | RESULTS AND DISCUSSIONS**

The discussions in this section are organised into four subheadings: a. Coexistence of spurious NPV and multiple IRR with loss-inducing investments; b. General criticism of no IRR, negative IRR, or zero IRR; b. Reinvestment of intermediate income; c. Resolution to multiple IRR and spurious NPV by eliminating reinvestment; d. Mutually exclusive investment cases; and e. IRR as a return on total investment and balance of capital.

**4.1 | Coexistence of Spurious NPV and multiple IRR with Loss incurring investments**

The historically famous ‘oil pump installation project’ data (see Table 1) studied by Solomon (1956:128) was estimated by several authors and all of them ended with multiple IRRs of 25% and 400% and a NPV of -\$774 and 0 at 10%, 25% and 400%, respectively. Kulakov and Kastro (2015) used the same data and concluded that the project’s rate of return was 16.9% (called it Generalized IRR (GIRR)). The DCF and MCAS methods estimated IRRs and NPVs for the oil pump case, and some other NDCF investments (with intermittent negative cash flow) are presented in Table 1.

a. Analysts would have rejected all four investments as they ended up with a loss of \$1600, \$100, and \$50 or zero profit (sum of NCF). That being the reality, the multiple IRRs (including GIR) and NPV (0 or positive) are all spurious. Spurious IRRs (multiple) and spurious NPVs coexist with such loss-making investments.

b. The positive NPVs for investments 3 and 4 (\$28 and \$33) are spurious. Had the NPV rule not been applied, these loss-making investments would have been wrongly selected.

**Table 1: Coexistence of Spurious NPVs and Multiple IRRs with Nonnormal NCF**

| Years   | 1. Solomon (1956:128) | 2. Excel user Microsoft | 3. Damodaran (2011) | 4. Arnold & Hope (1990) |
|---|-----------------------|-------------------------|---------------------|-------------------------|
| 0   | -1600                 | -1000                   | -1000               | -2000                   |
| 1   | 10000                 | 6000                    | 800                 | 5100                    |
| 2   | -10000                | -11000                  | 1000                | -3150                   |
| 3   | 0                     | 6000                    | 1300                | 0                       |
| 4   | 0                     | 0                       | -2200               | 0                       |
| <b>Sum of NCF (NPV at 0%)</b>                         | <b>-1600</b>          | <b>0</b>                | <b>-100</b>         | <b>-50</b>              |
| <b>NPV AND IRR ESTIMATED BY DCF METHOD</b>            |                       |                         |                     |                         |
| IRR 1   | 25%                   | 100.0%                  | 6.6%                | 5.0%                    |
| IRR 2   | 400.0%                | 200.0%                  | 36.5%               | 50.0%                   |
| IRR 3   | NIL                   | 0.0%                    | NIL                 | NIL                     |
| NPV at 10%  | <b>-\$774</b>         | <b>-\$128</b>           | \$28                | \$33                    |
| NPV at IRR1   | \$0                   | \$0                     | \$0                 | \$0                     |
| NPV at IRR2   | \$0                   | \$0                     | \$0                 | \$0                     |
| <b>NPV AND IRR ESTIMATED BY A MODIFIED CAS METHOD</b> |                       |                         |                     |                         |
| NPV at 10%  | -3636                 | -667                    | -157                | -207                    |
| IRR   | -100%                 | 0%                      | -9%                 | -2%                     |

1. Solomon (1956: 128) 3. Applied Corporate Finance, 3rd Edition 2. <http://www.exceluser.com/formulas/irr.htm> 4. Arnold & Hope (1990) c. As a test case, these NDCF were estimated using a MCAS method that eliminates the reinvestment, and the results are:

<sup>5</sup> CAS refers to the Capital Amortization Schedule and when the CAS is modified to allow for the elimination of reinvestment, the CAS is called a Modified CAS (MCAS).

- In all the four cases above, the unique IRRs are -100%, 0%, -9% and -2%, for the four NNCFs.
- These IRRs reflect the capital loss (loss as a percentage of the capital invested i.e., return on capital - ROC). For example, in the case of NCF 1 above, the loss was -1600 for a capital of 1600 and therefore 100% loss of capital; therefore, a unique IRR is -100%. Similarly, in the cases of NCF 2 to NCF4, the unique IRR are 0%, -9% and -2%, respectively. The profit or loss is a percentage of capital investments (ROC or IRR).
- The MCAS estimated NPVs also reflect the loss by each investment. For example, the positive NPVs estimated under the DCF method for NCF 3 and 4 are now negative after the elimination of the reinvestment by the MCAS method.

**4.2 | Reinvestment of intermediate income, Multiple IRR and Spurious NPV**

As rightly pointed out by Illés (2016) “the debate about the reinvestment rate assumption began in the 1950s (Solomon 1956; Renshaw 1957) and is still underway”. The most common assertion is that the IRR involves reinvestment at IRR and NPV at the hurdle rate. The validity of this assertion or assumption is investigated using both NCF and NNCF investments. The intermediate income from an investment is available to the investor to repay a loan, to meet other expenses, or to invest further. It is a decision to be made by the management and not by the estimation method. The reinvestment income should be eliminated from the NCF. The CAS method transparently reveals reinvestments, if any. The MCAS method eliminates the reinvestment and estimates the IRR and NPV. The results are furnished in Table 2, followed by a brief discussion. a. With normal NCF (investment A), the opening balances (OB) for all the years are negative, and therefore, there are only interest expenses and no interest income, or no reinvestment income is included. As there is no reinvestment, there is no difference between the IRRs (18.3%) and NPVs (\$183.9) estimated under CAS and MCAS methods. There is no reinvestment at IRR nor at the hurdle rate (10%) in the case of NPV (see Table 2). b. Another normal NCF (Investment B) is evaluated. The estimated IRR does not involve reinvestment, but the NPV of \$403 at 10% does involve reinvestment in years 3 and 4 (in bold numbers). When the MCAS method (that eliminates reinvestment; see years 3 and 4) is used, the NPV at 10% is \$376. This is an important finding that with normal NCF, the reinvestment does affect NPV in some cases but not the IRR. In such a case, the NPV is spurious, and the NPV rule is misleading.

**Table 2. An Analysis of Reinvestment of Intermediate Income at IRR or Hurdle Rate**

| Year  | 0      | 1      | 2      | 3           | 4          |
|---|--------|--------|--------|-------------|------------|
| <b>A. NCF Investment - CAS at IRR: 18.03% - No reinvestment</b>                         |        |        |        |             |            |
| NCF   | -1000  | 300    | 400    | 500         | 300        |
| Opening Capital   |        | -1000  | -880   | -639        | -254       |
| Interest/discount at 18.03%   | 18.03% | -180   | -159   | -115        | -46        |
| Income  |        | 300    | 400    | 500         | 300        |
| Closing Balance   |        | -880   | -639   | -254        | 0          |
| <b>A. NCF Investment - CAS at 10% NPV = \$184 (PV of CB**): \$269 - No reinvestment</b> |        |        |        |             |            |
| Opening Capital   |        | -1000  | -800   | -480        | -28        |
| Interest at Hurdle Rate: 10%  | 10%    | -100   | -80    | -48         | -3         |
| Income  |        | 300    | 400    | 500         | 300        |
| Closing Balance   |        | -800   | -480   | -28         | 269        |
| <b>B. NCF Investment - CAS at IRR: 31.7% - No reinvestment</b>                          |        |        |        |             |            |
| NCF   | -1000  | 600    | 600    | 300         | 200        |
| Opening Capital   |        | -1000  | -716.5 | -343.3      | -151.9     |
| Interest/discount at 31.70%   | 31.70% | -316.5 | -226.8 | -108.6      | -48.1      |
| Income  |        | 600    | 600    | 300         | 200        |
| Closing Balance   |        | -716.5 | -343.3 | -151.9      | 0          |
| <b>B. NCF Investment - CAS NPV at 10% : \$403 (PV of CB: 591) - With Reinvestment</b>   |        |        |        |             |            |
| Opening Capital   |        | -1000  | -500   | <b>50</b>   | <b>355</b> |
| Interest / discount at 10%  | 10%    | -100   | -50    | <b>-5</b>   | <b>-36</b> |
| Income  |        | 600    | 600    | 300         | 200        |
| Closing Balance   |        | -500   | 50     | 355         | 591        |
| <b>B. NCF Investment - MCAS - NPV at 10% : \$ 376 (PV of 550)</b>                       |        |        |        |             |            |
| Opening Capital   |        | -1000  | -500   | 50          | 350        |
| Interest at Hurdle Rate: 10%  | 10%    | -100   | -50    | <b>0</b>    | <b>0</b>   |
| Income  |        | 600    | 600    | 300         | 200        |
| Closing Balance   |        | -500   | 50     | 350         | 550        |
| <b>C. NNCF Investment - CAS at IRR: 26.9%</b>   |        |        |        |             |            |
| NNCF  | -50000 | 50000  | 30000  | -40000      | 30000      |
| Opening Capital   |        | -50000 | -13475 | 12894       | -23631     |
| Interest/discount at 26.9%  |        | -13475 | -3631  | <b>3472</b> | -6369      |
| Income  |        | 50000  | 30000  | -40000      | 30000      |
| Closing Balance   |        | -13475 | 12894  | -23631      | 0          |
| <b>C. NNCF Investment - CAS AT 10% : NPV at 10% : \$10685 (PV of the CB: \$15645)</b>   |        |        |        |             |            |
| Opening Capital   |        | -50000 | -5000  | 24500       | -13050     |
| Interest at Hurdle rate: 10%  |        | -5000  | -500   | <b>2450</b> | -1305      |
| Income  |        | 50000  | 30000  | -40000      | 30000      |
| Closing Balance   |        | -5000  | 24500  | -13050      | 15645      |
| <b>B. NNCF Investment - MCAS at IRR: 23.26%</b>   |        |        |        |             |            |
| Opening Capital   |        | -50000 | -11632 | 15662       | -24338     |
| Interest /discount at 23.26%  |        | -11632 | -2706  | 0           | -5662      |
| Income  |        | 50000  | 30000  | -40000      | 30000      |
| Closing Balance   |        | -11632 | 15662  | -24338      | 0          |
| <b>B. NNCF Investment - MCAS at 10% : NPV at 10% : \$8850 (PV of CB: \$12950)</b>       |        |        |        |             |            |
| Opening Capital   |        | -50000 | -5000  | 24500       | -15500     |
| Interest at Hurdle Rate: 10%  |        | -5000  | -500   | 0           | -1550      |
| Income  |        | 50000  | 30000  | -40000      | 30000      |
| Closing Balance   |        | -5000  | 24500  | -15500      | 12950      |

\*\* CB = Closing Balance

c. Under the NNCF (investment C), the OB is positive in year 3, which leads to reinvestment income (interest income or positive interest) included under the estimate. Including reinvestment income, the estimated IRR and NPV for the NNCF (B) are 26.9% and \$10685, respectively. Contrarily, the estimated IRR and NPV for the same NNCF (B) under the MCAS method (that excludes reinvestment income) are 23.26% and \$8850. This illustrates that with some NNCF investment, the IRR and the NPV are overestimated as they are equally affected by the reinvestment income. IRR and NPV, as estimated by the MCAS method, which excludes reinvestment, are the appropriate estimates. The assumption of reinvestment at IRR or at hurdle rate in NPV is false in the cases of normal NCF and some of the NNCFs. However, such a reinvestment is evident only with NNCFs that affect IRR and NPV.

In summary: a. with normal NCF investments, there is no reinvestment of intermediate income, neither at IRR nor at hurdle rate in the case of NPV; b. With some normal NCF investment analysis, reinvestment affects the NPV, not the IRR. c. With NNCF investments, there is reinvestment with some (not all<sup>6</sup>) of the NNCFs (see also Arjunan 2019); c. the reinvestment is a problem associated with some NNCF<sup>3</sup> data and has nothing to do with IRR or NPV, as assumed or asserted in most published works; and d. Any modification in the estimation of IRR or NPV, based on reinvestment assumption without evidence, is inappropriate. The resolution of multiple IRRs is discussed next.

**4.3 | Issues of no IRR, negative or zero IRR and multiple IRR**

Multiple IRR, no IRR, zero IRR and negative IRR are often criticised as a fatal flaw of IRR. Conceptually, if there are multiple IRRs, then the NPVs will also be zeros as many times as the number of IRRs. The problem of multiple IRR and corresponding multiple zero NPVs are caused by reinvestment associated with NNCF data. Similarly, the problem of ‘no or negative or zero’ IRRs is common with NNCF (rarely NCF) investments. The discussion in this section starts with the cases of “no or negative or zero IRR”. The results of the numerical analysis conducted are presented in table 3 and the results summarized here:

**Table 3. NCF and NNCFs - Estimated IRR and NPV by DCF/CAS and MCAS (Modified CAS) Methods**

| Details                     | NCF 1       | NCF 2       | NCF 3    |
|-----------------------------|-------------|-------------|----------|
| Year 0                      | 0           | -1000       | -1000    |
| Year 1                      | 1000        | 900         | 900      |
| Year 2                      | 1500        | -300        | -300     |
| Year 3                      | 2000        | 900         | 1000     |
| Year 4                      | 2500        | -600        | -600     |
| <b>Sum of NCF (SNCF)</b>    | <b>7000</b> | <b>-100</b> | <b>0</b> |
| IRR by CAS/DCF <sup>7</sup> | No IRR      | No IRR      | 0.0%     |
| NPV at 10% (CAS/DCF)        | \$5,359     | -\$163      | -\$88    |
| Reinvestment                | 846         | 900         | 600      |
| IRR by MCAS Method          | No IRR      | -7.4%       | 0.0%     |
| NPV at 10% (MCAS)           | \$4781      | -\$186      | -\$118   |

NCF 1 (see table 3) does not have any capital investments under the investment year (zero year). There is no need for the estimation of IRR or to recover capital invested (ROC) when there is no capital invested (i.e., no negative inflow). Naturally, there will be no IRR. Such NCF (with all positive cash flows) naturally leads to reinvestment of the intermediate income (\$846). There will be no IRR when the net cash flow stream is entirely positive or entirely negative. With NCF 1 data (with all positive cash flows), the estimated NPV by CAS or DCF method, which includes reinvestment income (\$846), is \$5359. For the same data, the estimated NPV by the MCAS method, which does not include reinvestment income, is \$4781. The NPV is spurious because the estimate includes reinvestment at the hurdle rate (cost of capital CoC). The NCF 2 is an NNCF, and the sum of NNCF is negative (-\$100). With such NNCF data, there is no IRR under the CAS or DCF method. A CAS analysis that includes reinvestment income of \$900 reveals that at zero interest rate, the CB is -\$100, the NPV is -\$100 at 0% and -\$163 at 10%. An MCAS analysis that excludes reinvestment indicates that the CB is zero at -7.4%, which means the IRR is -7.4%, and the NPV at -\$186. With such NNCF

<sup>6</sup> CAS of a NNCF investment that has positive OB in one or more years, only will lead to reinvestment.

<sup>7</sup> Estimates by CAS and DCF perfectly match (see Arjunan, 2019).

investments, both the IRR and NPV are spurious because the reinvestment income distorts them.

The undiscounted NCF is '0' in the case of NCF 3 (a NNCF). The IRR is 0% for NCF3, and the cash inflow completely offsets the capital and operating costs before discounting. Such situations are quite normal, and that does not mean IRR is an unsuitable criterion. The difference between the NPVs at 10% under CAS/DCF (-\$88) and MCAS methods (-\$118) is caused by the reinvestment income. The NPV is, therefore, spurious and cannot provide a better investment criterion in these cases. The IRR is consistently zero and truly reflects that the investment could not generate a return on invested capital (ROIC). In summary, these findings illustrate that those cases of 'no or zero or negative IRR' are the true reflection of the intrinsic value of the NCF and are consistent with the NNCF investments. Such NNCF, with reinvestments, leads to spurious NPV and IRR. The widely accepted "NPV rule" needs re-examination, given that NPV does suffer and is spurious with NNCF. The MCAS method resolves these problems; further analyses are discussed in the next sections.

**Multiple IRR:** As discussed, multiple IRR is mainly associated with NNCF, particularly if there is reinvestment of intermediate income. Projects A (Damodaran, 2010) and B (Chen 2008) with NNCF data, available in the public domain, are selected for this analysis. In both projects, the NNCF is negative at year 0 and again at year 4. Therefore, the following analysis is presented to clarify the impact of reinvestment on IRR and NPV and resolve the multiple IRR problem. The results are presented in Table 4 and summarised here:

**Table 4. NNCFs and Estimated IRRs and NPVs by CAS/DCF and MCAS Methods**

| Year              | NNCF A | Estimates     | By CAS/DCF | By MCAS  |
|-------------------|--------|---------------|------------|----------|
| 0                 | -1000  | IRR (1)       | 6.6%       | -9.0%    |
| 1                 | 800    | IRR (2)       | 36.5%      | NA       |
| 2                 | 1000   | NPV at 6.6%   | 0          | -\$142.2 |
| 3                 | 1300   | NPV at 36.5%  | 0          | -\$193.4 |
| 4                 | -2200  | NPV at 10%    | \$27.8     | -\$157.1 |
| Sum of NCF (SNCF) | -100   | NPV at 20%    | \$52.5     | -\$183.3 |
|                   |        | NPV at -9%    | -\$396.3   | 0        |
| Year              | NNCF B | Estimates     | By CAS/DCF | By MCAS  |
| 0                 | -580   | IRR (1)       | 9.9%       | -12.1%   |
| 1                 | 530    | IRR (2)       | 32.2%      | NA       |
| 2                 | 530    | NPV at 9.9%   | 0          | -\$94.3  |
| 3                 | 530    | NPV at 32.2%  | 0          | -\$109.0 |
| 4                 | -1080  | NPV at 10%    | \$0.4      | -\$94.8  |
| Sum of NCF (SNCF) | -70    | NPV at 20%    | \$15.6     | -\$105.7 |
|                   |        | NPV at -12.1% | -\$319.8   | 0        |

The sum of undiscounted NCF (SNCF) for projects A and B is -\$100 million and -\$70 million, respectively. In both cases, there is a net loss, or the net income is negative even without discounting, and no commercial investor will consider such investments.

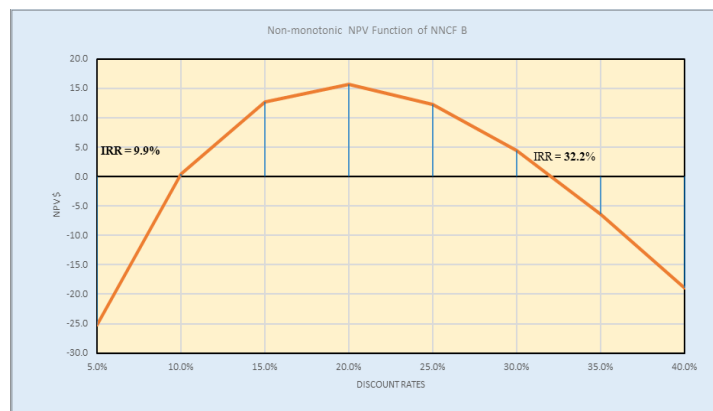
*IRRs and NPVs estimated by CAS/DCF method*

For projects A and B, there are two IRRs of 6.6% and 36.55% (9.9% and 32.2%)<sup>8</sup>The NPVs are also zero twice, once at 6.6% and again at 36.55% (9.9% and 32.2%). For an assumed hurdle rate of 10%, the NPVs at 10% for projects A and B are positive \$27.80 and \$0.4, respectively. When both projects are loss-making, i.e., the sum of NCFs is negative \$100 and \$70, respectively, the positive NPV is spurious. When the NPV is zero at discount rates 6.6% (9.9%), getting a positive NPV of \$52.47 million (\$15.6 million) at 20% is spurious. Such a spurious value of NPV (increases with higher discount rates) is typical of non-monotonic NPV function, normally associated with NNCF (see Arjunan 2019; Merlo 2012). The usual discounting using the hurdle rate (a point estimate) fails to capture the spurious NPV dynamics. Both IRRs and NPVs are spurious, and therefore, it is not rational to argue that IRR suffers with multiple rates (two here) without acknowledging the fact that NPV does suffer with a multiplicity of spurious NPVs caused by the NNCF.

*IRRs and NPVs estimated by MCAS method*

An MCAS method (discussed under methodology) eliminates reinvestment of income, resolves the problem of multiple IRR, and leads to a unique IRR. The results derived by the MCAS method are discussed here:

With elimination of the reinvestment income, the IRRs are unique at -9.0% and -12.1%, for project A and B, respectively. The negative IRR realistically reveals an approximate loss of 9% or 12% (of the capital invested) and therefore these investments must be rejected. The NPVs at 10% are also negative in both projects A and B, under MCAS method. With the current practice of DCF or CAS methods, had the 'NPV rule' or 'IRR rule' been adopted, both A and B would have been accepted as the NPVs at 10% are positive, and at least one of the IRRs is above the hurdle rate. The result indicates capital loss, and therefore, the NPV rule is misleading, and the IRR is confusing. Chart 1 presents the non-monotonic function of the NNCF B (Chen 2008). The NPVs for discount rates below 9.9% (IRR 1) are all negative. NPVs are positive for discount rates above 9.9% to 32.2% (IRR 2) and are zero at IRR1 and IRR 2. The highest NPV was recorded at a 20% discount rate. The NPVs are all spurious, and the IRR is not unique. The MCAS method leads to a unique IRR of -12.1% that reflects around 12% capital loss (loss of 70/580). The NPV rule leads to a wrong decision (positive NPV at 10%).



**Chart 1: Non-monotonic function of NNCF B (Chen 2008)**

In summary, both IRR and NPV suffer from multiple estimates with NNCF data, but then the criticism is equally, rather more, applicable to NPV. As pointed out earlier, the DCF or CAS analysis using unorthodox or non-normal NCF, with negative or zero or negligible undiscounted value, affects the credibility or rationality of the analysis. The limitation of the input data (NNCF) and the DCF/CAS method must be clearly understood. The MCAS method, for NNCF and NCF, offers consistent and realistic estimates and resolves the problem of multiple IRR and spurious NPV.

**4.4 | Mutually Exclusive Projects**

Investment decision, to accept or reject or rank mutually exclusive projects, based on contradicting NPV or IRR criteria is another controversial issue. The validity of this argument is critically evaluated in this section. IRR and NPV are estimated for two sets of mutually exclusive projects at various discount rates, including an assumed hurdle rate of 10% and at IRRs of both projects (the IRR of project A is used as a discount rate for project B and vice versa). The results are furnished in Table 5 in two parts: 1. NCFs for two sets of mutually exclusive projects and the estimated IRR and NPV; and 2. NPV profiles of mutually exclusive projects.

**Table 5. Estimated IRR, NPV and NPV profiles**

| Years / Criteria   | Silber (2016) – Set I |                  | Damodaran (2010) – Set II |                |
|--|-----------------------|------------------|---------------------------|----------------|
|  | Project A             | Project B        | Project A                 | Project B      |
| <b>1. NCFs for the two sets of mutually exclusive projects</b> |                       |                  |                           |                |
| 0  | -1000                 | -1000            | -1000                     | -10000         |
| 1  | 0                     | 320              | 350                       | 3000           |
| 2  | 0                     | 320              | 450                       | 3500           |
| 3  | 300                   | 320              | 600                       | 4500           |
| 4  | 600                   | 320              | 750                       | 5500           |
| 5  | 900                   | 320              | -                         | -              |
| IRR  | 14.70%                | 18.00%           | 33.70%                    | 20.90%         |
| NPV \$ at 10%  | 194.03                | 213.05           | 653.13                    | 2757.33        |
| <b>2. NPV Profiles of mutually exclusive projects selected</b> |                       |                  |                           |                |
| Silber (2016)  |                       | Damodaran (2010) |                           |                |
| Discount Rate  | Project A: NPV        | Project B: NPV   | Discount Rate             | Project A: NPV |
| 10.00%   | 194.03                | 213.05           | 10.00%                    | 653.13         |
| 14.70%   | 0                     | 81.2             | 15.00%                    | 467.94         |
| 15.00%   | -12.23                | 72.69            | 20.00%                    | 313.08         |
| 18.00%   | -115.5                | 0                | 20.90%                    | 288.37         |
| 20.00%   | -175.35               | -43              | 30.00%                    | 71.2           |
|  |                       |                  | 33.70%                    | 0              |
|  |                       |                  |                           | -2188.64       |

<sup>8</sup> Figures in parenthesis correspond to project B.

Based on the results presented in Table 5, had the point estimate (at cost of capital) NPV (or NPV rule) been used, projects B and A under the first and second sets of projects, respectively, would have been accepted. However, a review of the NPV profiles reveals:

Under set I, project B is supported by the highest NPV for various discount rates and is zero at 18% and negative for discount rates above 18%. The NPV of project A is zero at 14.7% (IRR) and negative for all discount rates above 14.7%. Project B, with the highest NPV at higher discount rates than Project A, must be logically selected. Chart 2 presents the NPV functions of projects A and B of set I. As could be seen, Project B recorded higher NPV for discount rates above 14.7% and recorded higher IRR and should be selected. Under set II, the NPV profiles indicate that project 'A' has a higher NPV at discount rates below 15% and reaches zero at 33.7% 20.9% (IRR of A) and negative for discount rates above 33.7% 20.9%. Project B has a higher NPV for discount rates below 15% and reaches zero at 20.9% (IRR of B). A rational investor will prefer project 'B' with a higher IRR because with an increasing discount rate up to the IRR level, the NPV will also be higher with project 'A' and not with project 'B'. The IRR reveals the potential profitability of the investments and is, therefore, a better criterion than NPV. The NPV is a point estimate valid for a particular discount rate.

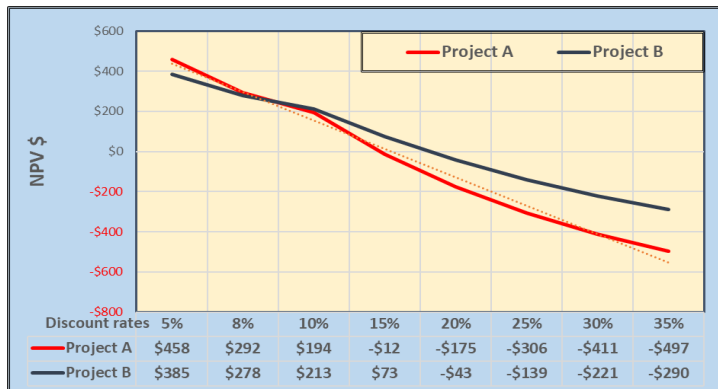


Chart 2. Mutually Exclusive Projects Ranking (e.g., Silber 2021)

Project A - IRR =15% and Project B IRR =18%  
Ranking by NPV vary with discount rates whereas ranking by IRR is unique

Based on this analysis, it is concluded that the NPV rule must be revisited, and the IRR is a better criterion than NPV to rank mutually exclusive and independent investments.

#### 4.5 | IRR as return on total capital vs return on declined balance of capital

The CAS method enables estimation of IRR on the declining capital or total capital as furnished in Table 6. The only change is to estimate interest or apply the discount rate for the initial capital investment rather than the reduced opening balance of the capital. As could be seen, the IRRs are 24.9% and 20% on declining capital (normal IRR) and the total capital invested, respectively. With return on total capital, the interest income or return is constant for all years (see CAS, interest row in Table 6). The important point is that the declining capital is due to the recovery of part of the capital every year. The recovered capital can be invested elsewhere to generate a return (opportunity cost of capital). That income will compensate for the difference between the return on the declining balance and the total capital.

Table 6. IRR as Return on Declining Balance of Capital vs Total Invested Capital

|                          |                       |        |         |         |         |       |
|--------------------------|-----------------------|--------|---------|---------|---------|-------|
| NCF: CAS A               |                       | -10000 | 3000    | 4000    | 5000    | 6000  |
| Opening Capital          |                       |        |         |         |         |       |
|                          |                       | -10000 | 9488.8  | 7850.4  | 4804.3  |       |
| Interest at 24.9%        | On balance of capital |        |         |         |         |       |
|                          |                       | 2488.8 | -2361.6 | -1953.8 | -1195.7 |       |
| Income                   |                       | 3000.0 | 4000.0  | 5000.0  | 6000.0  |       |
| Closing Balance          |                       |        |         |         |         |       |
|                          |                       | 9488.8 | 7850.4  | 4804.3  | 0.0     |       |
| IRR on Declining Capital |                       |        |         |         |         | 24.9% |
| NCF: CAS B               |                       | -10000 | 3000    | 4000    | 5000    | 6000  |
| Opening Capital          |                       |        |         |         |         |       |
|                          |                       | -10000 | 9000.0  | 7000.0  | 4000.0  |       |
| Interest at 20%          | On total capital      |        |         |         |         |       |
|                          |                       | 2000.0 | 2000.0  | 2000.0  | 2000.0  |       |
| Income                   |                       | 3000.0 | 4000.0  | 5000.0  | 6000.0  |       |
| Closing Balance          |                       |        |         |         |         |       |
|                          |                       | 9000.0 | 7000.0  | 4000.0  | 0.0     |       |
| IRR on Total Capital     |                       |        |         |         |         | 20.0% |

## 5 | SUMMARY AND CONCLUSION

In this paper, the validity of the NPV rule and common criticism against IRR is evaluated. The main findings are summarized here below:

Multiple IRR and Spurious NPV Coexist: With NNCF data, both IRR and NPV suffer from multiple IRR and spurious NPV. The results indicate that the NNCF data and the DCF method are the causes, and the effects are spurious multiple IRR and NPV. The MCAS method eliminates the reinvestment income and thereby resolves the problem of spurious NPV and multiple IRR, leading to a unique IRR.

Reinvestment of intermediate income: The present analytical evidence indicates no reinvestment with normal NCF and some of the NNCF data. This finding is important and rejects the reinvestment assumption at IRR or hurdle rate. In some normal NCF cases, the reinvestment affects the NPV and makes it spurious, but not the IRR.

No or zero or Negative IRR: When the NCF does not have a single negative flow (in year 0 or 1), there will be no IRR. In such cases, the NPV collapses into the PV of benefits. Similarly, if the sum of undiscounted NCF is 0 or negative (no profit or net loss), then there is no IRR or negative IRR, which truly reflects the intrinsic value of the NCF.

Mutually Exclusive Investments: Among the mutually exclusive projects, the NPV rule prefers investments with the highest NPV at the hurdle rate. The NPV rule ignores the higher NPV achieved by higher discount rates. A review of the NPV profile at various discount rates will reveal the highest NPV that could be achieved. Evidence indicates that investments with higher IRR achieve higher NPV at higher discount rates than their counterparts with higher NPV at hurdle rates. This finding rejects the conventional wisdom that only NPV can rank mutually exclusive investments. IRR explores and estimates the potential maximum return and can rank the investments better.

Return on Total capital: The CAS method enables the estimation of the IRR as a return on total capital and on the balance of capital.

In conclusion, the results indicate that the NPV rule is not always valid, and the criticisms against IRR are not always valid. As the CAS approach resolves the problems with CBA and perfects the IRR and NPV estimates, investment criteria IRR (return as a %) or NPV (return in \$ terms) or the profitability Index or BCR (Return per \$ of investment) can be used by the decision makers according to their preference. All three criteria should lead to the same decision when the CAS method estimates.

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